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Strategic Uncertainty as a Cause of War*

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ABSTRACT

This paper shows why states, acting in their own self-interest, may create informational asymmetries that lead to war. In our models, two actors with no private information invest in military capacity before engaging in crisis bargaining. If bargaining fails, the states go to war, and the payoffs of a war depend on the two states' military capacities. We examine a large class of models and show that states have incentives to keep each other guessing about their exact levels of military capacity — even though doing so creates the risk of war. Thus, self interest and strategy are to blame for the emergence of uncertainty about military strength and war. Our paper explains two stylized facts: States devote considerable resources to secrecy in the national-security realm, and often disagree about the balance of capabilities.

INTRODUCTION

Many rationalist explanations for war hinge on private information or uncertainty. While the standard argument stems from a simplified description of the international environment, its logic is quite compelling. By most accounts, war is costly both in terms of weapons and lives; it uses up resources and is, thus, inefficient.As a consequence, we may think of international disputes as concerning the division of a pie that shrinks in the

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event of a war. At least in principle, there is a peaceful settlement that all states prefer to a war — a settlement in which each state gets at least the share of the pie it would have gotten in war, and at least one gets more. If each state knew others' values for war precisely, the states would be able to reach such a peaceful settlement. Thus, a hypothetical world in which states possess all relevant information is completely peaceful.

Unfortunately, this peaceful description is inaccurate; states hide information about their militaries, not only about specific programs but also about their overall military budgets and the strength of their armed forces. For example, the United States Department of State publishes a book containing estimates of the foreign military expenditures of most foreign countries, and it does so with this qualification:

> A primary aim [of the document being quoted] is to inform the reader of the main qualifications to the data, much of which is not as accurate as uniform presentation in statistical tables may imply. *This is particularly true of the data on military expenditures, armed forces, and arms transfers, which in many countries are subject to severe limitations of incompleteness, ambiguity, or total absence due to governmental secrecy* (emphasis added).[1]

If the United States, with its large budget and extensive intelligence network, has trouble obtaining reliable information about other states' military expenditures and armed forces, imagine the trouble faced by other countries.[2]

Fearon's (1995) well-known argument shows that extant private information (like that observed by the State Department) can have pernicious consequences. When private information is present, states have incentives to misrepresent their values for war. For this reason, they may fail to reach a bargain and go to war, even though doing so is inefficient (war reduces the size of the pie).[3]

But why are states uncertain about important attributes of their potential opponents in the first place? Fearon's argument is not wholly satisfying because it does not explain the emergence of private information. Instead, it takes as a premise that states begin their interaction with private information and explains why asymmetries of information are maintained throughout negotiations. The assumption that states begin with private information is unproblematic insofar as this asymmetric information is truly exogenous. However, insofar as states are uncertain about attributes that other states control (e.g., military capacity), understanding why states make choices that lead to uncertainty helps us to understand the origins of military conflict more completely.

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1 "Statistical Notes," downloaded from Department of State web site: http://www.state.gov/t/vci/rls/rpt/wmeat/1999_2000/.

2 In some cases the State Department also may have better estimates than it acknowledges, or other branches may have better estimates than they share with the State Department.

3 Private information plays an important role in many theories of war. For example, in Powell's (1999) model of bargaining in the shadow of power, states never go to war with complete information; they may, however, go to war when one has private information. Blainey (1973) writes about mutual optimism as a cause of war. For an argument against mutual optimism as an explanation for war, see Fey and Ramsay (2007).

The present paper helps to explain the emergence and persistence of asymmetric information about military capacity.[4] More specifically, we show why states (i) arm, (ii) do so in a manner that is not predictable and (iii) keep their capacity secret, even though doing so involves a risk of war. To be clear, Fearon's and others works speak to point (iii) — keeping the capacity a secret, given that (i) and (ii) are descriptively accurate. In this paper, we take a step back and also investigate why states choose to arm and why their armament decisions remain unpredictable, as in the above excerpt, when it is known that this unpredictability will create the possibility of inefficient war. Our explanation for war has an important normative implication: self-interest and strategy can lead parties to undertake actions that make war possible even when they start in a world without asymmetric information, which one might think would be a world of peace.

We find that states create and keep secrets about their overall level of military capacity and risk war unless there is no net benefit to either state from arming secretly and unilaterally and starting a war, relative to the settlement that reflects the status-quo levels of military capacity. Why do states create uncertainty about military capacity, even though doing so creates a risk of war? States have two competing incentives when it comes to acquiring military capacity. First, insofar as states' military capacities are known, a state with greater military capacity is likely to do better in negotiations; if a war occurs the capacity also will be beneficial (Banks 1990). Military capacity is costly, however, so states do not wish to acquire too much; this incentive is especially strong if negotiation is likely to lead to a peaceful settlement. In fact, when peaceful settlement is anticipated, there is a very clear incentive to minimize the actual investment in capacity but pretend that one has made a large investment (though this deception does not happen in equilibrium).

In a strategic setting, these individual preferences translate into a conundrum. If each state knows that an adversary is unarmed, then they will reach a settlement. But if both are unarmed, then it is likely that either can gain from unilaterally increasing its capacity and engaging in a surprise attack — a unilateral increase in strength makes victory much more likely and this offsets the cost of acquiring additional capacity as well as any costs of fighting. If two states' arms and their levels of strengths are known, they again can reach a bargain that is mutually preferred to war; however, as long as the states expect to reach a bargain, either benefits from secretly investing in a lower level of capacity. As a consequence of these competing incentives, we show that states' decisions about how much military capacity to accumulate in our model are unpredictable (they are in mixed strategies), except when the incentive to arm unilaterally is absent, a condition we discuss later. Following Palfrey and Rosenthal (1985), we call the resulting uncertainty "strategic."[5]

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[4] The literature on international conflict identifies several factors that influence a state's resolve (Morrow 1989a, Fearon 1992). We follow Morrow (1989a) in focusing on military capacity; Morrow explains the effects of uncertainty about capacity, while we focus on its origins.

[5] Readers may recall that there are two forms of strategic uncertainty. Players may not observe the choices of other players, i.e., the game involves imperfect information, and players may not know the costs, strengths or preferences of other players, i.e., the game involves incomplete information.

In practice, states create "strategic uncertainty" (make their adversaries uncertain about their exact level of capabilities) by arming somewhat unpredictably and by keeping military secrets. Note that keeping purchases or expenditures secret is not in itself enough to create uncertainty about a state's military strength — some degree of unpredictability also is necessary. For example, if a state's equilibrium strategy were to buy five tanks with certainty, even adversaries that could not observe the purchase would not be uncertain about the state's number of tanks. Thus, when we refer to states "keeping secrets" or "hiding information" in this paper, we mean this expanded version of secrecy, accumulating military strength in such a way as to make it possible for their exact levels of strength to be secret, and then not revealing this exact level of strength.[6]

Of course, we do not mean to say that states act completely unpredictably in the colloquial sense of that phrase, that next year's military budget in the United States could be anything. Military budgets often, though not always, change incrementally, and much information about military acquisitions is public in democracies. Rather, we mean to say that Secretaries of Defense, Presidents, and their counterparts elsewhere make decisions about building their military forces in such a way that adversaries do not know the country's overall level of strength precisely, and then they keep secrets to preserve this uncertainty. For example, countries often divide their military expenditures among several departments, in part to make it harder to figure out the budget precisely. In the United States, these include the Department of Energy and in Russia the atomic-energy ministry. Leaders also keep particular programs secret. While there are multiple reasons for states' secrecy about military matters, one consequence is that adversaries have difficulty in estimating their overall strength with precision.

By explaining the strategic origins of states' uncertainty about their adversaries' overall military strength, our paper provides a foundation for the literature on war that assumes the existence of private information. It also explains the corresponding fact that states often lack information about each other's military forces. In equilibrium, two states with the same information cannot disagree about the balance of military forces. Thus, our work also provides an equilibrium explanation for why states often disagree about the military balance, as was the case at the start of the Russo-Japanese War (Fearon 1995, pp. 398–400). Finally, our paper speaks to the literature on international institutions, which argues that institutions can reduce the risk of war by providing members with information, and thus reducing the uncertainty (e.g., Keohane (1984, pp. 93–95)). This argument again presumes that states have private information in the first place. Moreover, our analysis explains why it is difficult to build institutions that reduce the scope of states' uncertainty about security (see Keohane (1984, pp. 94, 247)). If institutions were to do

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Our paper purposely considers a game in which the second form of strategic uncertainty is not present and focuses on the first type of strategic uncertainty.

6 States in our models keep secrets by not revealing their acquisitions through bargaining; while we do not analyze cheap talk in this paper, it is easy to see that they still keep secrets if given the opportunity to talk.

away with informational asymmetries, states still would have the incentives we identify to increase their capacity and to do so privately (that is, to create uncertainty).

Our results about the difficulties of avoiding uncertainty and the ensuing risk of war are quite general; we show that they hold for many bargaining protocols and under different assumptions about states' ability to observe or learn about each other's military investments. States in each of our models begin with complete information. We begin by examining models in which states do not observe their adversaries' investments in military capacity directly. We show that a version of our results holds for any bargaining protocol satisfying two very reasonable conditions: (1) under the protocol, if the parties were to know each other's capacities, they would reach a bargain short of war in equilibrium, and (2) under the protocol, it is possible for either party to initiate a war without the consent of the other state (although this need not occur in equilibrium). That is, we show that despite the fact that states would reach a settlement if they maintained the complete and perfect information they have at the start of the game, they choose to create uncertainty and to generate a risk of war.

We next show that our substantive conclusions hold as long as purchases of military capacity are even *slightly* difficult to observe directly (rather than completely unobserved). If states can demonstrate their military capacities precisely (and cannot cheat by secretly acquiring more), then equilibria without war do exist under a wider range of circumstances; in some of these circumstances, states arm and in others, they do not. However, if states' observations of or other information about their adversaries' military acquisitions are even a little noisy, then in equilibrium they create uncertainty and generate a risk of war under the same conditions as in our original model, subject to a weak assumption about when states can unilaterally initiate conflict.

These results about states' ability to observe each other's military acquisitions again speak to the effects of international institutions. If institutions provide information that is even slightly imperfect and states are sufficiently patient, then states create uncertainty and risk war despite the presence of the institution.

Our model differs from most previous formal models of crisis bargaining in that it begins with complete information (any uncertainty is endogenous) and has a rich bargaining space with no commitment problems, and yet in equilibrium states sometimes go to war. Most previous models of bargaining in international crises have been either complete-information models in which states never go to war (e.g., Fearon (1995), Kydd (2000), Powell (1999)), or incomplete-information models in which states begin the game possessing private information and go to war in equilibrium (e.g., Fearon (1995), Morrow (1989a), Powell (1999), Slantchev (2005), Schultz (1998)). Powell (1993) and Morrow (1989b) contain complete-information models in which states go to war; in these models states do not have the opportunity to reach a bargain short of war.[7] In the sense that it predicts that wars occur with rich bargaining protocols and no *ex*

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[7] Some authors have created incomplete-information models in which states do not go to war. For example, in Kydd (2000), an arms race resolves the underlying uncertainty, so the states do not go to war in equilibrium. Levontoglu and Tarar (2008) show that under some circumstances private information results in delay in reaching a settlement rather than in war.

ante asymmetric information, our paper is similar to Slantchev (2003). However, our explanation for war is quite different. Slantchev shows that in the presence of multiple, stage-game equilibria and an infinite horizon, war can be supported in the short run by threats of playing inefficient equilibria in subsequent periods.

Our paper is most closely connected to contemporaneous work by Jackson and Morelli (2007), who analyze a dynamic game in which states select levels of investment and decide whether or not to fight in each period. In contrast to the current paper, Jackson and Morelli assume that investment decisions are perfectly observed. For some assumptions about the costs of war, they find equilibria in which states randomize in their investments and eventually go to war. In these equilibria, the forces that lead to randomization in the investment stage are similar to the forces that lead to randomization in the current paper and are closely connected to contests in general. Their finding that war can occur after perfectly observed investments, however, depends on aspects of the dynamic structure in their game and differs from our findings in the section with observed investments. Jackson and Morelli do not study a situation in which investment decisions are imperfectly observed. Our paper also is related to Baliga and Sjöstrom (2008). In their model, private information is more exogenous than in ours; weak states that try to acquire weapons of mass destruction (WMD) may or may not succeed in doing so, leading to uncertainty over whether or not they have WMD. However, when states decide to preserve this uncertainty, they do so to balance the costs and benefits of arming, analogous to incentives in our model.

The paper develops as follows. In the next section, we present the basic model of arms accumulation in the setting of a stylized assumption about the bargaining protocol. We characterize necessary and sufficient conditions for equilibria in which states always avoid war in this model. Subsequently, we generalize the results to a very large class of bargaining protocols. We then turn to issue of equilibrium existence, and illustrate that the logic of the second section applies to a large class of models for which equilibria are known to exist (possibly in mixed strategies). In the penultimate section, we relax the assumption that states' military purchases are not directly observed. In the last section, we conclude.

THE MODEL

Before turning to the models and analyses, we make a few clarifications about the modeling strategy and terminology. States in each of our models begin with complete information. In this section and in the next section, we examine models in which states do not observe their adversaries' investments in military capacity directly.(In later sections of the paper, we relax this assumption about unobserved investments, to rather surprising results.) In allowing states to take hidden actions to acquire capacity, however, we are not simply assuming that states have private information. In an equilibrium in which the states' capacity investments are in pure strategies, each state can figure out the capacity of its opponent and the ensuing equilibrium bargaining corresponds to equilibrium behavior in a bargaining model with complete information. This feature is standard in Nash equilibria. Equilibria of our game typically involve mixed strategies at the capacity-accumulation stage — that is, each state has a probability distribution over the levels of capacity that it buys. As long as this is the case, after the states acquire military capacity, each state has beliefs about the other's chosen capacity, but knows its own level precisely. Thus, states face uncertainty about their opponents' levels of military capacity. In other words, states begin the interaction with symmetric information, but we show that in most settings they generate asymmetric information, or uncertainty, in every equilibrium of the game.

This paper does not attempt to provide a complete explanation for how states make decisions about arming, negotiating, and fighting. Instead, we provide very general answers to questions about the circumstances under which states' behavior leads to uncertainty about their military capabilities, and those under which they are able to avoid war with certainty. A cost of our focus on generality is that we do not answer a natural, follow-up question: Precisely what happens in equilibrium?

In the model, two states begin by simultaneously investing in military capacity. Each state's investment is unobserved by the other player, and each pays a per-unit cost for capacity. After investing in capacity, the states bargain; if bargaining fails, they go to war.

For simplicity's sake, we assume that the states bargain over a good of size one that does not begin as the property of either state. Either the states agree on a division of the pie that totals one, or they go to war. Our analyses would not be affected by changing the size of the good or by assuming that one or the other state began by owning the good.

Formally, we consider two states, 1 and 2. It is often convenient to refer to them as i and j. In period 1, each state simultaneously chooses an investment $m_i \geq 0$ in military capacity.[8] Only state i observes the choice of m_i. Investment by i has a marginal cost $\beta_i > 0$. In period 2, the states bargain. (In later extensions of the game, bargaining may last many periods; in this case, they begin bargaining in period 2.) Either the states agree to a settlement a_i, a_j such that $a_i + a_j = 1$ and $a_i, a_j \geq 0$, or they engage in military conflict. In the event of a conflict, the payoff to state 1 is given by the function $w_1(m_1, m_2)$ and the payoff to state 2 is $w_2(m_2, m_1)$. By $\mathbf{m}$, we denote a vector of investment levels. When it creates no ambiguity, we write $w_i(\mathbf{m})$.

Two assumptions are imposed on the payoffs to war. First, military investment by country i can only help country i in the event of war, and it can only hurt the adversary, j. Second, war is inefficient relative to a peaceful bargain. That is, following Fearom (1995) and others, we assume that war is costly; people die, equipment is destroyed or used up, and territory may be devastated. This means that the total pie available to be divided is decreased if the states go to war. Formally, these assumptions are:

Assumption 1 For $i \in \{1, 2\}$ the function $w_i(\mathbf{m}) : \mathbb{R}^2_+ \to [0, 1]$ is nondecreasing in m_i and nonincreasing in m_j with $j \neq i$.

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[8] We use the terms capacity and arms interchangeably in this paper, but capacity in the model can include any factors that make a state more likely to win a war but are costly to accumulate — for example, a new technology or military strategy.

Assumption 2 $w_1(\mathbf{m}) + w_2(\mathbf{m}) < 1$ for all $\mathbf{m}$.[9]

To begin, we focus on a very simple bargaining process, or protocol: State 1 makes an offer $a_1 \in [0, 1]$ and state 2 either accepts it (getting $1 - a_1$) or rejects it. Following a rejection, the states fight a war. This is the protocol analyzed in Fearon (1995). Later in the paper, we show that the main results hold for a large class of bargaining protocols. In this simple version of the model, we sometimes refer to the first state as the proposer and the second as the veto player. We focus on perfect Bayesian equilibria (PBE), which require that at each information set play is sequentially rational given beliefs and beliefs are updated according to Bayes' Rule whenever possible.

For simplicity, we consider two states that start the game possessing capacity of $a_i = 0$. An exercise in relabeling allows the game to capture a situation in which the states begin the game with known levels of capacity other than 0. In this case, as we discuss later, the choices a_i can be interpreted as net enhancements in capacity, and 0 as each state's status-quo level of capacity.

Of course, in practice there may be reasons for investing in military capacity that are unrelated to relations with the adversary — for example, bureaucratic politics. Insofar as there is randomness (or asymmetric information) in these other investments, then there are other sources of uncertainty than those that we identify.

It is also likely that investments in military capacity that affect only relations with the adversary in our model also have spillover effects in practice. For instance, investments might support a segment of the economy, satisfy constituencies, or provide for security in ways unrelated to the dyad that the game focuses on. One way to reinterpret the model in this context is to recognize that these additional factors decrease the marginal cost of investment (relative to the other things a state can do with resources). Since the marginal cost is a parameter, β_i, such an interpretation is entirely consistent with our analyses and it is easy to see how changes in these additional benefits might change the results, as we discuss later in the paper.

Results

We first consider the types of pure strategy equilibria that are possible — that is, under what circumstances are there equilibria in which states' actions do not lead to uncertainty? As we discussed earlier, states in our model begin with complete information about all features of the game. Since, in equilibrium, strategies are common knowledge, both states will continue to know all of the relevant information as long as they acquire military capacity in pure strategies.

Since our model is one of *ex ante* complete information, one might expect a number of equilibria in which states arm but reach bargains short of war. We begin by showing that no such equilibrium exists. To do so, we begin by showing that states do not arm in

[9] Most formal studies treat war as a costly lottery, with each state's probability of winning and thus its expected utility reflecting the distribution of power (Wagner 2000, Powell 2004). This approach satisfies Assumptions 1 and 2, so our results apply to models with this payoff structure.

any pure strategy equilibrium of this game. The reason for this is that states will reach a bargain short of war in any pure strategy equilibrium, since, as long as states accumulate military capacity in pure strategies, the capacity levels are known (though not observed) at the time of bargaining. However, if states are avoiding war with certainty, each prefers to secretly remain unarmed, since acquiring arms is costly. Thus, if states are reaching a bargain and $m_i > 0$, state i has an incentive to deviate to $m_i = 0$.

Proposition 1 *There are no equilibria with pure strategy accumulations and* $\max\{m_1, m_2\} > 0$.

Proof: Suppose that there is a pure strategy equilibrium with $m_i > 0$ for some i. In the bargaining stage of the game, sequential rationality requires that state 2 accept any offer that satisfies $1-a_1 \geq w_2(\mathbf{m})$. This conclusion relies on the fact that in games of this form it is not possible to support equilibria in which the second mover rejects offers when she is indifferent. An acceptance strategy of this form results in an open-set-problem for the proposer and thus equilibrium requires a selection of the correct rule for how players act when indifferent; we do not belabor this point in the remainder of the paper. Thus, the maximum offer a_1 that will be accepted is $1 - w_2(\mathbf{m})$. An offer by state 1 above this amount will be rejected by state 2, which leads to the war payoff $w_1(\mathbf{m})$. By assumption (2), $1-w_2(\mathbf{m}) > w_1(\mathbf{m})$, so sequential rationality requires that 1 offer $1-w_2(\mathbf{m})$, which is accepted. Therefore, war does not occur in this equilibrium and thus an unobserved deviation to $m_i = 0$ will not change the outcome in the bargaining stage, but it will increase i's payoff by $\beta_i m_i$.Thus, this is a profitable deviation. ■

Thus, in any equilibrium in which states do not acquire uncertainty, neither state accumulates any military capacity.

We next show that an equilibrium in which neither state arms and therefore states remain certain about each other's capabilities is the only possible equilibrium in which the states never go to war.[10] Put differently, states go to war with positive probability in any equilibrium in which they generate uncertainty. We also show that an unarmed, certain-peace equilibrium exists only under a very limited set of circumstances. The intuition behind this next result is again simple: Because states do not directly observe each other's military capacity, each country can arm heavily without the other knowing. Thus, a situation in which both states are unarmed and always at peace is hard to sustain, because each has an incentive to become strong secretly and start a war.

To find the weakest condition that supports an equilibrium in which neither state acquires military capacity ($\mathbf{m} = \mathbf{0}$), we first assume that such an equilibrium exists. In such an equilibrium, the offer must be $a_1 = 1 - w_2(0, 0)$ as this maximizes player 1's payoff while keeping player 2 indifferent between the war and the offer. This equilibrium requires that neither party is willing to unilaterally deviate from $m_i = 0$. Since $\beta_i > 0$, such a deviation is only worthwhile if war occurs (with positive probability) following the deviation.There are two such deviations that must be ruled out: (1) The veto player unilaterally deviates (accumulating a positive amount) and rejects a_1. This

[10] The following result also implies that there is no equilibrium in which only one state plays a mixed accumulation strategy (generates uncertainty) and war occurs with probability 0.

deviation is worthwhile only if, $w_2(m_2, 0) - \beta_2 m_2 > w_2(0, 0)$. (2) The proposer deviates by accumulating capacity and offering less to the veto player.This is only worthwhile if $w_1(m_1, 0) - \beta_1 m_1 > 1 - w_2(0, 0)$.

Proposition 2 *War occurs with probability zero in a particular equilibrium if and only if* $\mathbf{m} = \mathbf{0}$ *with probability one in the equilibrium. Moreover, an equilibrium of this type exists if and only if*

$$w_2(m_2, 0) - \beta_2 m_2 \leq w_2(0, 0) \quad \textit{for all } m_2 \in (0, \infty) \tag{1}$$

and

$$w_1(m_1, 0) - \beta_1 m_1 \leq 1 - w_2(0, 0) \quad \textit{for all } m_1 \in (0, \infty). \tag{2}$$

Proof:

Step 1: We first establish the first part of the proposition. Suppose that there is an equilibrium in which war happens with probability 0. This means that on the path the offer is accepted. Since $\beta_i > 0$ a deviation from any lottery putting probability on $m_i > 0$ to another strategy with $m_i = 0$ with probability one would be desirable. Thus, $\mathbf{m} = \mathbf{0}$ must occur with probability 1 in the equilibrium. Assume that $\mathbf{m} = \mathbf{0}$ with probability 1 in equilibrium. Given this and Assumption 2 it is well known that in all perfect equilibria of the ultimatum game a settlement is reached.

Step 2: We now establish that such an equilibrium occurs if and only if the stated condition is satisfied. Given $\mathbf{m} = \mathbf{0}$, sequential rationality in bargaining requires that the proposer, 1, offer $a_1 = 1 - w_2(0, 0)$ as long as $1 - w_2(0, 0) \geq w_1(0, 0)$. This condition is true by assumption. There are two deviations to consider: a deviation by the proposer, 1, to increase m_1 and make an offer which is not accepted or a deviation by the veto player, 2, to increase m_2 and to reject the offer. If state 2 uses the above threshold strategy then 1's best possible deviation is desirable iff (2) is not satisfied. Alternatively state 2's deviation is desirable iff (1) is not satisfied. ■

A certain-peace equilibrium in which states do not hide information can exist — but only under a particular set of circumstances (parameterizations). When we take the model literally, assuming that states that do not purchase arms in the model are completely unarmed, these circumstances seem likely to occur very rarely in the real world. Proposition 2 states that the certain-peace equilibrium exists only when it is better to be a weak state negotiating with another weak state than a heavily armed state at war with an unarmed adversary.

To give some sense of the difficulty in obtaining a certain-peace equilibrium if we interpret the baseline level of arms as zero, we turn to an example with specific payoffs for war. One might think of many wars as contests: the state that has the greater military capabilities wins, and takes the spoils of war, while the other state loses, and gets nothing. Our next example reflects this idea: In the event of war, the more heavily armed state wins and gets the prize; the other state loses and gets nothing, and the states split the prize if they have acquired equal military capacity. Formally, for a number $\alpha \in (0, 1)$

we can define

$$w_j(m_j, m_i) = \begin{cases} \alpha & \text{if } m_j > m_i \\ \dfrac{\alpha}{2} & \text{if } m_j = m_i \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

In this winner-take-all example, the conditions of Proposition 2 are satisfied if and only if $\alpha = 0$. Thus, in this example, the certain-peace equilibrium exists only if war provides no goods to the victor.

We would argue, however, that the correct interpretation of Proposition 2 is less pessimistic than this example would suggest because states have some (at least partially) known, baseline level of military capabilities when they make their decisions about armaments. If this is the case, then the certain-peace equilibrium exists as long as each state's net payoff to secretly arming as much as it likes and starting a war is less than its payoff from the status-quo bargain, or the bargain that reflects the status-quo distribution of power. This condition is more likely to be satisfied because states sometimes are quite happy with the status-quo bargain and therefore may not find it worthwhile to pay the costs of secretly arming and launching a war.[11]

While one might think that states will be more likely to hide information about their capacity when military capacity is more expensive, Proposition 2 states that this is not the case. All else equal, the conditions for the certain-peace equilibrium are more likely to hold when the cost parameters are higher, because secretly arming is more costly. Unfortunately, the interpretation of Proposition 2 therefore is more pessimistic if we believe that military spending has positive externalities that are unrelated to the dyadic relationship (e.g., bolstering the economy, pleasing domestic constituencies), which can be thought of as lowering the cost parameter in the model.

In sum, this section has shown that in every equilibrium of the game, states act somewhat unpredictably and keep secrets about their military capabilities, thus risking war, unless each state derives no net benefit from war in which it is heavily armed relative to the status quo. They do so even though the "strategic uncertainty" that accompanies this behavior carries with it a risk of war. Empirically, a certain-peace equilibrium in which states do not act unpredictably seems more plausible if states have some known level of military forces acquired for other reasons, but less so if military expenditures driven by the adversarial relationship have positive externalities.

OTHER BARGAINING PROTOCOLS

We have seen that, when bargaining is characterized by the ultimatum game, states create uncertainty about military capabilities and risk war if either condition (1) or condition

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[11] If each state begins with a known, status-quo level of capabilities, q_i, then the relevant condition is as in Proposition 2, but with each 0 replaced with q_i. While we have assumed very little about a state's payoff from war, a reasonable assumption might be that the war payoff function is concave (exhibiting diminishing returns) in the country's own investment so that each state has greater incentive to secretly buy more arms if both begin unarmed.

(2) is not satisfied. We now investigate whether this result extends to a larger class of bargaining protocols. We find that it remains the case that if states do not create uncertainty and there is a zero probability of war in any equilibrium then the states must remain completely unarmed (or maintain the status quo levels of military capacity) in that equilibrium. Moreover, in all of these models, rather stringent conditions (which represent natural extensions of conditions (1) and (2)) must be satisfied in order for certain-peace equilibria to exist.

The results in this section apply to all bargaining protocols that meet two conditions. First, as we discussed at the start of the paper, as long as war is costly, there is always some settlement that both states prefer to war. When states have complete information, they can identify settlements of this form. Thus, we consider bargaining protocols under which states reach an efficient settlement (one that fully divides the pie) if they have complete information. This is a large class of protocols; it includes protocols with veto players and the alternating-offer Rubinstein bargaining model that often is used to study international conflict. Second, we would argue that any state eventually has the possibility of opting out of bargaining and starting a war without the consent of its adversary, though it may never choose to do so. If instead both states have to consent to war, a state with military capacity could be forever stuck in a bargaining process it does not like, with no option of unilaterally using whatever forces it possesses. Thus, we assume that either state can end the bargaining and start a war, unilaterally and in a finite amount of time. The first condition is a bit more subtle as it restricts protocols based on particular aspects of their equilibrium set; the second condition is more straightforward as it speaks only to the structure of the protocol and not any description of how states will actually behave in the protocol. We now give formal definitions.

Formally, the bargaining protocol includes a description of the available offers, sequence of speaking, and how behavior maps into payoffs. Specifically, we define a bargaining protocol to be a pair of strategy spaces, S_1, S_2 and a mapping $b(\xi_1, \xi_2)$: $S_1 \times S_2 \to A \times \mathbb{N}$. The strategy spaces may be finite, countably infinite, or uncountably infinite. The set A is defined to be the union of the two-dimensional simplex and a particular outcome that we call war. We denote this outcome by the vector $(-1, -1)$. Specifically,

$$A := \{(p_1, p_2) \in \mathbb{R}^2_+ : p_1 + p_2 = 1\} \cup \{(-1, -1)\}. \quad (4)$$

The mapping $b(\xi_1, \xi_2) \mapsto (p_1(\xi_1, \xi_2),\ p_2(\xi_1, \xi_2),\ t(\xi_1, \xi_2))$ assigns a vector of allocations to the two states and a time $t \in \mathbb{N} := \{1, 2, \ldots\}$ at which the decision is reached for any profile of bargaining positions. If $(-1, -1)$ is the outcome and it is reached in period t, then we say that bargaining breaks down in period t and war occurs. The payoffs from this outcome are given by $\delta^{t-1} w_i(m_i, m_j)$. If a settlement is reached in period t then state $i's$ payoff is $\delta^{t-1} p_i(\xi_1, \xi_2)$. The term $\delta \in (0, 1]$ serves as the common discount factor.[12] Since we are not modeling the details (timing and information sets) in the bargaining protocol, we use a normal-form-game refinement instead of PBE. Thus, we

[12] We are assuming that bargaining costs can be represented adequately by the discount factor — that is, if settlement is delayed by a period, each state gets a smaller payoff in present value, which is similar to paying a cost.

require that strategies form a trembling hand perfect equilibrium of the investing and bargaining game. Because trembling hand perfect equilibria are sequential equilibria, this requires that strategies in the bargaining histories are sequentially rational to beliefs about investments and that the beliefs are consistent.

In order to state the assumptions rigorously, we need to be able to talk about $\mathbf{m}$ being known. We say the accumulation $\mathbf{m}$ is known at the beginning of the bargaining protocol if player 1's belief about player 2's accumulation is concentrated at the correct value, player 2's belief about player 1's accumulation is concentrated at the correct value, and both of these facts are common knowledge. The first assumption states that when the accumulations are known, sequentially rational play results in a peaceful settlement.

Condition 1 (war is inefficient): Whenever $\mathbf{m}$ is known, every profile of sequentially rational play in the bargaining protocol results in some bargaining solution (p_1, p_2, t) that satisfies the conditions that $p_1 + p_2 = 1$ and t is finite.

The second assumption ensures that war is not a consensual outcome; either party can in a finite period of time initiate a conflict (the outcome $(-1, -1)$) regardless of the other player's bargaining behavior. This is an assumption about what is possible, and not about what occurs in equilibrium; neither state need ever start a war.

Condition 2 (war is not consensual): There exists a finite t' such that for either $i \in \{1, 2\}$ there exists a strategy $\xi_i' \in S_i$ s.t. for any $\xi_j \in S_j$, $b(\xi_i', \xi_j) = (-1, -1, t)$ for some $t < t'$.

The ultimatum bargaining protocol of the previous section does not satisfy Condition 2, since player 1 must make an offer and in principle player 2 could play the strategy: accept any offer. However, it is easy to see that if the ultimatum game were modified so that player 1 could just start a war instead of making an offer, the equilibrium set would not change in any important ways.

In the large class of bargaining models that satisfy these two conditions, we find, again, that in any equilibrium in which war occurs with probability zero, the states remain unarmed with probability one. For the class of bargaining models, certain-peace equilibria exist only when a condition exists that is analogous to the one we presented earlier. We begin by showing that states create uncertainty (accumulate military capacity in mixed strategies) in any equilibrium unless the states remain completely unarmed. That is, we generalize Proposition 1.

Proposition 3 *There are no equilibria with pure strategy accumulations satisfying* $\max\{m_i, m_j\} > 0$.

Proof: Assume that such an equilibrium exists with efforts $\mathbf{m}$. Since the equilibrium is in pure strategy accumulations, states possess all relevant information when they begin bargaining. By Condition 1, a bargain will be reached with probability one. Since effort is not observed, a deviation to $m_i = 0$ (and no change in i's bargaining actions) would not affect the outcome of the bargaining protocol. Since $\beta_i > 0$, for a state with $m_i > 0$ such a deviation would increase its utility. ■

The next result establishes that the states do not go to war if they remain completely unarmed. However, if states choose to arm and create uncertainty about their military capacities, there is always a positive probability that they go to war. This result rules out equilibria in which states generate uncertainty (accumulate military capacity in mixed strategies), but always reach a peaceful settlement through negotiation. It generalizes the first half of Proposition 2.

Proposition 4 *War occurs with probability 0 in an equilibrium if and only if* $\mathbf{m} = \mathbf{0}$ *with probability one in this equilibrium.*

Proof: We proceed in steps.

Step 1. If the players play pure accumulation strategies, then $\mathbf{m}$ is known. Condition 1 then implies that war does not occur.

Step 2. Suppose that there is an equilibrium in which war happens with probability 0. This means that on the path some offer a_i is accepted. Since $\beta_i > 0$ a deviation from any nondegenerate lottery to $m_i = 0$ (and no change in i's bargaining behavior) would be desirable. Thus, if we are in an equilibrium and no deviation is desirable it must be the case that $\mathbf{m} = \mathbf{0}$. ■

Finally, it is possible to present partial analogs to the necessary and sufficient conditions for the existence of the unarmed, certain-peace equilibria (1) and (2). We have assumed that the states reach a peaceful bargain if they have complete information. We show next that — as with the particular bargaining protocol we considered earlier — a sufficient condition for the certain-peace equilibrium is that neither state can benefit from unilaterally arming and starting a war when the other state is unarmed. Establishing necessary conditions for the unarmed, certain-peace equilibrium is a bit more challenging. In principle, the value to a state of deviating from its certain-peace equilibrium strategy, arming, and starting a war can be less than its full value for war (net of the costs of arming) because of the costs of delay. We have limited our consideration to bargaining protocols that allow either state to start a war, but not necessarily immediately. Thus, with some protocols in the class we are considering, if a state deviates by arming and starting a war, it will receive only a discounted payoff from war. This implies that even when the sufficient conditions for the certain-peace equilibrium are not satisfied, neither state may wish to deviate from its certain-peace equilibrium strategy of remaining unarmed. Let Π^0 denote the set of expected payoffs pairs that are feasible given some sequentially rational profile of bargaining behavior when it is known that neither state has accumulated any military capacity ($\mathbf{m} = \mathbf{0}$).[13]

Proposition 5 (i) *There is an equilibrium with* $\mathbf{m} = \mathbf{0}$ *if*

$$w_j(m_j, 0) - \beta_j m_j \leq \pi_j \quad \text{for all } m_j \in (0, \infty) \tag{5}$$

and

$$w_i(m_i, 0) - \beta_i m_i \leq \pi_i \quad \text{for all } m_i \in (0, \infty). \tag{6}$$

for some $(\pi_i, \pi_j) \in \Pi^0$.

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13 Recall, a payoff is the discounted value of an outcome. Thus each coordinate of a payoff vector in this set is of the form $\pi_i = \delta^{t-1} p_i(\xi_i, \xi_j)$.

(*ii*) *There is an equilibrium with* $\mathbf{m} = \mathbf{0}$ *only if*

$$w_j(m_j, 0) - \frac{\beta_j m_j}{\delta^{t'-1}} \leq \frac{\pi_j}{\delta^{t'-1}} \quad \textit{for all } m_j \in (0, \infty) \tag{7}$$

and

$$w_i(m_i, 0) - \frac{\beta_i m_i}{\delta^{t'-1}} \leq \frac{\pi_i}{\delta^{t'-1}} \quad \textit{for all } m_i \in (0, \infty). \tag{8}$$

for some $(\pi_i, \pi_j) \in \Pi^0$

Proof: (i) By assumption, following $\mathbf{m} = \mathbf{0}$ the payoffs correspond to the right-hand side of the inequalities for some equilibrium selection. We now consider a unilateral deviation by i. Given that $(\pi_i, \pi_j) \in \Pi^0$ the most that i can get from a peaceful settlement is π_i. If this were not true then (π_i, π_j) would not be an equilibrium payoff to bargaining when it is known that $\mathbf{m} = \mathbf{0}$. This implies that the deviation to $m_i > 0$ is profitable only if the deviation results in a lottery that puts positive probability on war and it would be nice not to break the math expression $w_i(m_i, 0) - \beta_i m_i > \pi_i$. This is not possible if $w_i(m_i, 0) - \beta_i m_i \leq \pi_i$ for all $m_i \in (0, \infty)$. Thus, the result is established.

(ii) Step 1: First, note that for every pair $(\pi_i, \pi_j) \in \Pi^0$ and the supporting equilibrium strategy profile, given knowledge of $\mathbf{m} = \mathbf{0}$, there is no unilateral deviation by i from the conjectured equilibrium strategy that involves $m_i = 0$ and different actions in the bargaining game that results in a payoff higher than π_i. Second, consider a unilateral deviation from a particular profile that starts with $\mathbf{m} = \mathbf{0}$ and has sequentially rational play in the bargaining protocol. Any such deviation that (i) reaches a settled outcome (not $(-1, -1)$) with probability one and (ii) involves $m_i > 0$ could be improved upon by the strategy that has $m_i = 0$ and mimics this deviation. These two points imply that in considering equilibria with $\mathbf{m} = \mathbf{0}$ it is sufficient to consider deviations that reach war with positive probability and yield war payoffs that exceed π_i.

Step 2: Assume that an equilibrium with $\mathbf{m} = \mathbf{0}$ exists. The conclusion of step 1 implies that it must be the case that for the t' defined in Condition 2

$$\delta^{t'-1} w_i(m_i, 0) - \beta_i m_i \leq \pi_i \quad \text{for all } m_i \in (0, \infty). \tag{9}$$

If this were not true, then the equilibrium with $m = 0$ would not exist as i could unilaterally increase its payoff by selecting some $m_i > 0$ and selecting the bargaining action ξ_i' defined in Condition 2. Since t' is finite, multiplication by $\frac{1}{\delta^{t'-1}}$ is permissible and the conclusion is established. ■

In order to contrast this result with conditions (1) and (2), consider bargaining protocols in which Π^0 is a singleton. The last result demonstrates that

$$w_i(m_i, 0) - \beta_i m_i \leq \pi_i \quad \text{for all } m_i \in (0, \infty) \tag{10}$$

is sufficient but not necessary for certain-peace equilibria. Recall that with the particular bargaining protocol considered earlier in the paper, this condition is necessary and sufficient. Within the larger class of bargaining protocols that we consider here, the necessary and sufficient conditions become analogs to conditions (1) and (2) if the states become sufficiently patient. (They converge to the natural analogs of conditions (1) and (2) in the limit as δ goes to 1.) Similarly, if the states have the option of starting a war immediately, though they may choose not to exercise it (that is, if $t' = 1$), then the certain-peace equilibrium conditions are exactly analogous to conditions (1) and (2).

Like Propositions 1 and 2, this result shows that states are less likely to hide information when arming is costly, because they are more likely to remain completely disarmed, or armed only at the known status-quo level. Looking at the left-hand side of the inequalities, one can see that the payoff to unilaterally arming and starting a war goes down as the per-unit cost of arming (β) goes up; thus states are more content to remain peacefully disarmed.

Within this large class of bargaining models, it also remains the case that states risk war whenever they arm in a way that creates uncertainty. Moreover, if states are sufficiently patient or have the option of starting a war immediately, the certain-peace equilibrium exists only if there is no incentive to secretly arm, unilaterally and perhaps heavily, and to start a war, thus overturning the status-quo bargain. If states are impatient, the existence of the certain-peace equilibrium depends upon the states' degree of patience and/or on the time that it takes them to mobilize forces and begin a war.

EXISTENCE OF EQUILIBRIA

In light of our focus on understanding the circumstances in which states choose not to create uncertainty, a natural question emerges. Do our models have equilibria in which states *do* create uncertainty — that is, equilibria in mixed strategies? If we assume that the set of possible investment levels and the strategy spaces in the bargaining protocol are finite (albeit arbitrarily large), then Selten (1974) guarantees that the answer is "yes"; trembling hand perfect equilibria exist in mixed strategies. We now discuss how the main results of the previous section hold in finite versions of the games we present here.

Consider a setting in which the set of armaments is a finite set $M_i \subset \mathbb{R}_+$ with $0 \in M_i$ (and $M_j \subset \mathbb{R}_+$ with $0 \in M_j$) and in the bargaining protocols, S_i and S_j are finite. Assume that Conditions 1 and 2 are satisfied.[14] In this context, Propositions 3–5 hold and the only change to the proofs is that phrases like $m_i \in (0, \infty)$ are replaced by phrases like $m_i \in M_i$.

Proposition 6 *Suppose that* $M_i \subset \mathbb{R}_+$, $M_j \subset \mathbb{R}_+$ *are both finite with* $0 \in M_i$ *and* $0 \in M_j$, *and the bargaining protocol is such that* S_i *and* S_j *are finite.*

(i) *This game possesses a* (*possibly mixed strategy*) *mixed strategy trembling hand perfect equilibrium.*

[14] Note that the assumption of finite strategy spaces in the bargaining protocol does not limit the possible bargains in a substantively meaningful way. Since the set of possible armaments also is finite in these versions of the games, the bargaining space can be much larger than the armament space. Thus, it is still reasonable to assume that with complete information, states will reach a bargain short of war.

(ii) *There are no equilibria with pure strategy accumulations satisfying* $\max\{m_i, m_j\} > 0$.
(iii) *War occurs with probability 0 in an equilibrium if and only if* $\mathbf{m} = \mathbf{0}$ *with probability one in this equilibrium.*
(iv) *There is an equilibrium with* $\mathbf{m} = \mathbf{0}$ *if*

$$w_j(m_j, 0) - \beta_j m_j \leq \pi_j \quad \text{for all } m_j \in M_j \tag{11}$$

and

$$w_i(m_i, 0) - \beta_i m_i \leq \pi_i \quad \text{for all } m_i \in M_i. \tag{12}$$

for some $(\pi_i, \pi_j) \in \Pi^0$.
(v) *There is an equilibrium with* $\mathbf{m} = \mathbf{0}$ *only if*

$$w_j(m_j, 0) - \frac{\beta_j m_j}{\delta^{t'-1}} \leq \frac{\pi_j}{\delta^{t'-1}} \quad \text{for all } m_j \in M_j \tag{13}$$

and

$$w_i(m_i, 0) - \frac{\beta_i m_i}{\delta^{t'-1}} \leq \frac{\pi_i}{\delta^{t'-1}} \quad \text{for all } m_i \in M_i \tag{14}$$

for some $(\pi_i, \pi_j) \in \Pi^0$.

Thus, we know that each finite version of the games we have discussed must have an equilibrium in which states create uncertainty and go to war with positive probability, unless the conditions for a certain-peace equilibrium are satisfied. In light of this result, we see that the nonexistence of certain-peace equilibria when conditions (13) and (14) are not satisfied should not be attributed to a technical pathology of the games that make equilibrium itself poorly defined. Instead, the nonexistence of certain-peace equilibria can be attributed to an incentive problem that leads countries to create and keep secrets about their military capabilities.

Establishing existence of mixed strategy equilibria in the infinite versions of the games we examine is nontrivial. Recall that a mixed strategy equilibrium involves both lotteries over investments and investment-contingent bargaining strategies; the investment lotteries must solve a fixed-point problem that is induced by the bargaining strategies, and the bargaining strategies must be sequentially rational (given beliefs). As such, the existence problem seems substantially harder than the already challenging one in Jackson and Morelli (2007).

ACQUISITIONS OF MILITARY CAPACITY CAN BE OBSERVED

Thus far, we have considered a large class of models in which states do not observe each other's military acquisitions directly, but could choose to maintain complete information by buying military capacity in pure strategies. We have shown the difficulties inherent in avoiding the creation of uncertainty about military capacity and an associated risk of war. It is easy to see that these difficulties do not disappear if states can engage in cheap talk after they acquire arms.

But do states act somewhat unpredictably in acquiring military capacity even when they can demonstrate their military acquisitions or observe their opponents'? We now consider two alternative extensions of our models in which states can observe each other's military acquisitions to varying degrees in order to address this question. These extensions build on the model with the general bargaining protocols.

Perfect Signals about Military Acquisitions

In the first model, states can choose whether or not to demonstrate their military acquisitions to their adversaries; if a state decides to do so, the adversary learns the information precisely (and the state cannot secretly acquire additional capacity). This model also can represent a situation in which a state can choose to allow an international institution to provide an adversary with information about its military capacity and the institution has the technology to acquire and provide this information perfectly.

As in our earlier models, the states begin by selecting their capacities, m_i, m_j, simultaneously. Now, however, each state also selects a message $s_i \in \{m_i, \phi\}$ prior to the bargaining process. If a state chooses $s_i = m_i$, this signal informs the adversary completely about the state's military acquisitions. (One can think of the message as a verified statement or a perfect demonstration.) If the state chooses $s_i = \phi$, it is deciding not to allow the adversary to observe its military acquisitions. After the messages, the states bargain; if bargaining fails, they go to war. In this subsection, we assume that bargaining occurs according to the ultimatum bargaining protocol, as in the first model in this paper.

In this setting, a state cannot deviate from a conjectured equilibrium strategy profile that involves informative communication without letting its opponent know that it has deviated. The requirements for a pure strategy equilibrium $m = (m_1, m_2)$, $s = (m_1, m_2)$ are (a) that

$$\begin{aligned} m_1 &\in \arg\max_m \{1 - w_2(m_2, m) - \beta_1 m\} \\ m_2 &\in \arg\max_m \{w_2(m, m_1) - \beta_2 m\}, \end{aligned} \tag{15}$$

and (b) that given the strategy profile no player has an incentive to deviate to a pair m_i', ϕ with $m_i' \neq m_i$.

To see the possibility of pure strategy equilibria, we consider two examples. The first follows other works in the international-relations literature (e.g. Powell (1999), Chapter 2) in assuming that a state's payoff from war is increasing in the share of the two states' forces that it possesses:

$$w_i(m_i, m_j) = \begin{cases} \dfrac{pm_i}{m_i + m_j} & \text{if } (m_i, m_j) \neq (0,0) \\ \dfrac{p}{2} & \text{if } (m_i, m_j) = (0,0). \end{cases} \tag{16}$$

With $p \in (0, 1)$, first-order conditions from (a) yield

$$\begin{aligned} \frac{pm_2}{(m_1 + m_2)^2} &= \beta_1 \\ \frac{pm_1}{(m_1 + m_2)^2} &= \beta_2 \end{aligned} \tag{17}$$

and the solution is given by

$$\begin{aligned} m_1 &= \frac{p\beta_2}{(\beta_2 + \beta_1)^2} \\ m_2 &= \frac{p\beta_1}{(\beta_2 + \beta_1)^2}. \end{aligned} \tag{18}$$

These values satisfy the second-order conditions at any values of m_1 and m_2. So as long as the payoffs to these accumulations exceed 0, condition (a) is satisfied. The boundary condition requires

$$\begin{aligned} \frac{p\beta_2}{p\beta_2 + p\beta_1} &\geq \frac{p\beta_1\beta_2}{(\beta_2 + \beta_1)^2} \\ \frac{p\beta_1}{p\beta_2 + p\beta_1} &\geq \frac{p\beta_1\beta_2}{(\beta_2 + \beta_1)^2}. \end{aligned} \tag{19}$$

Simplifying yields

$$1 \geq \max\left\{\frac{p\beta_1}{\beta_2 + \beta_1}, \frac{p\beta_2}{\beta_2 + \beta_1}\right\}. \tag{20}$$

In order to check that condition (b) is satisfied, we need to specify off-the-path beliefs for bargaining games in which $s_i = \phi$. The equilibrium is supported if the beliefs assign probability 1 to $m_i = 0$ given $s_i = \phi$. To see this, note that we already have seen that the equilibrium payoff to i exceeds the payoff to bargaining as if $m_i = 0$. It remains to verify that i does not prefer to arm, announce $s_i = \phi$, and then have a conflict. For country 1, the maximum payoff to this deviation solves

$$m_1 \in \arg\max\left\{\frac{pm_1}{m_1 + m_2} - \beta_1 m_1\right\}, \tag{21}$$

which has the same first-order condition as above since the partial derivatives of $1 - \frac{pm_2}{m_1+m_2}$ and $\frac{pm_1}{m_1+m_2}$ with respect to m_1 are the same. Since the second state is solving the same problem, the optimal deviation cannot improve its payoff. It is interesting to note that in this equilibrium, secrecy is interpreted as a sign of weakness. That is, when states reveal information about their capacities in equilibrium, but have the option of choosing not to, opponents must treat failure to reveal information as a signal that i is weaker than the equilibrium level m_i. Were this not true, i would benefit from hiding its information (changing s_i from m_i to ϕ). The results of our analysis of this example are as follows.

Proposition 7 *With the war technology in* (16) *and the ability to demonstrate capacity perfectly prior to play of the ultimatum game*, *there is an equilibrium in which the states select* *capacities in pure strategies, reveal their capacities, and reach a negotiated settlement; war occurs with probability* 0.

Is the beneficial effect of perfect monitoring limited to the particular military technology that we have just considered? Earlier in the paper, we considered a situation in which war payoffs are like a winner take all contest. With these payoffs, the game also has pure strategy equilibria in which the states avoid war completely.

To see the possibility of pure strategy equilibria with the war payoffs given in (3), note that an equilibrium of the form $m = (m_1, m_2)$, $s = (m_1, m_2)$ must result in a tie or at least one player must be selecting $m_i = 0$. In either of these cases, one might think that a player could improve its payoff by selecting a slightly higher accumulation than the opponent and announcing it. However, equilibria of this sort can exist because a country is willing to accumulate at most $m_i = \frac{\alpha}{\beta_i}$ in order to win a prize worth α, or $\frac{\alpha}{2\beta_i}$ in order to tie. Thus, for example, when $\beta_i = \beta_j$, there is a pure strategy equilibrium in which both states accumulate $\frac{\alpha}{2\beta_i}$ and avoid war.

Overall, the results in this section seem encouraging. While they rely upon a particular bargaining protocol, a more general version could be developed that allows for a large class of protocols. In addition, the results from the first example (in which the payoff from war is increasing in the state's share of the military forces) can be readily extended to necessary and sufficient conditions for pure strategy equilibria under more general differentiable payoff functions. Extensions of this form are technical. Instead, we conclude the paper with an important and surprising robustness check.

Noisy Signals about Military Acquisitions

The previous section showed the benefits of being able to demonstrate military capacity perfectly: If statements about military acquisitions are known to be correct, states may arm and yet always avoid war. However, in the real world, technologies for monitoring others' military capacity and thus verifying their claims about their capabilities are imperfect. For this reason, we now consider a model in which states obtain information about their opponent's choices of military capacity, but this information is imprecise. This model, which extends our treatment of general bargaining protocols differs in two ways from the one in the previous section. First, a state does not choose whether or not to give information about its capacity to the opponent; instead, the opponent exogenously receives a signal. Second, more importantly, the signal a state receives about its adversary is noisy.

If instead of demonstrating one's strength, ($s_i = m_i$) states only can observe a noisy signal of strength, $s_i = m_i + \varepsilon$, where ε might be thought of as white noise, the encouraging findings of the previous section do not generalize. No matter how small the variance of ε, if the shocks have full support and players are sufficiently patient then a certain-peace equilibrium exists only under strong conditions like those presented in part (ii) of Proposition 5. Thus, there is a disjuncture between complete demonstration of capacity, which we considered in the previous section, and noisy signals, which we consider here. Even a little bit of noise in the information a player gets about its opponent's capacity is enough to make war a possibility.

We now assume that after the states have simultaneously chosen their levels m_1 and m_2, but before they bargain, each state i receives an exogenous noisy signal $\sigma_i \in \mathbb{R}^1$ about the other's (j's) chosen capacity. In the event that a state plays a mixed strategy, the opponent receives a noisy signal about the realization of the mixed strategy. We assume nothing about the distribution of the signal $F(\sigma_i|m_j)$ except that it has full support for any level of capacity m_j that the state may have chosen and that conditional on m_i, m_j the signals are independent.[15] The full support assumption states that for two distinct levels m_j' and m_j'' the conditional distributions $F(\cdot|m_j{}')$ and $F(\cdot|m_j{}'')$ have the same support. This assumption requires that a state cannot completely "rule out" any level of capacity because of the signal it has observed. This assumption can be satisfied in settings in which the conditional distributions have arbitrarily low variance. For example (though this need not be the case), it could be that the distribution of the signals that i receives is normally distributed with a mean of m_j and variance $\varepsilon > 0$, where ε is very small.

The reason why many of the results about the difficulties of avoiding uncertainty and war hold with partial observability is that the addition of the noisy signals $\boldsymbol{\sigma} = (\sigma_1, \sigma_2)$ does not change the fact that if states acquire capacity in pure strategies, deviations will not be detected even though the signals are observed. More technically, if state j acquires $m_j = m_j^1$ with probability one in equilibrium, then by Bayes' Rule, state i must believe that $m_j = m_j^1$ with probability one after seeing any σ_j, since the prior belief is that $m_j = m_j^1$ and $F(\sigma_j|m_j)$ has full support for all m_j.

Why are the cases of perfect and imperfect observability different? With perfect observability, any deviation from a pure strategy equilibrium is immediately known. With the form of imperfect observability that we investigate here, a state cannot observe when its adversary has deviated. Thus, with imperfect observability it often is easier to find profitable deviations and therefore more difficult to support equilibria in pure strategies.

We begin again by showing that states create uncertainty (accumulate military capacity in mixed strategies) in any equilibrium unless the states remain completely unarmed and that states do not go to war if they remain completely unarmed.

Proposition 8 (i) *There are no equilibria in which the accumulations are in pure strategies with* $\max\{m_i, m_j\} > 0$. (ii) *In any equilibrium in which the accumulations are in pure strategies, war occurs with probability 0.*

Proof: (i) Assume that such an equilibrium exists with efforts $\{m_i^*, m_j^*\}$. Since the equilibrium accumulations are in pure strategies and $F(\sigma_i|m_j)$ has full support for all m_j, states possess all relevant information when they begin bargaining. By condition 1, a bargain will be reached with probability one. Since state j must believe that $m_i = m_i^*$ after seeing any σ_j, a deviation to $m_i = 0$ would not affect the outcome of the bargaining protocol. Since $\beta_i > 0$, for a state with $m_i^* > 0$ such a deviation would increase its utility.

[15] It would be natural to assume that the distributions $F(\cdot|m_j{}')$ and $F(\cdot|m_j{}'')$ are ordered (in terms of likelihood ratio or first order stochastic dominance) if $m_j' < m_j''$. Our analysis is, of course, consistent with this type of assumption, but this structure is not needed for the following results.

(ii) Consider an equilibrium with pure strategy accumulations. Since the equilibrium is in pure strategy accumulations and $F(\sigma_i|m_j)$ has full support for all m_j, by Bayes' Rule, i continues to believe $m_j = m_j^*$ after seeing any σ_i and j continues to believe $m_i = m_i^*$ after seeing any σ_j. Condition 1 then implies that war does not occur. ■

Finally, we show that if states generate uncertainty in equilibrium, there is always a positive probability that they go to war. It is most direct to prove the contrapositive of this claim.

Proposition 9 *If war occurs with probability 0 in an equilibrium that involves no delay*[16] *or in any equilibrium to a game with* $\delta = 1$, *then* $\mathbf{m} = \mathbf{0}$ *with probability one in this equilibrium.*

Proof: We proceed in steps.

Step 1: We consider an equilibrium with pure strategy accumulations. Suppose that there is an equilibrium in which both states' accumulations are in pure strategies, at least one state's accumulation is not zero ($m_i \neq 0$), and the states go to war with probability 0. This means that on the path some offer a_i is accepted with probability 1. Since the equilibrium involves pure strategy accumulations, by Bayes' Rule, i continues to believe $m_j = m_j^*$ after seeing any σ_j and j continues to believe $m_i = m_i^*$ after seeing any σ_i. Since $\beta_i > 0$, a deviation to $m_i = 0$ would be desirable. Thus, if we are in an equilibrium with pure strategy accumulations and no deviation is desirable, it must be the case that $\mathbf{m} = \mathbf{0}$.

Step 2: Now we consider an equilibrium in which player i's accumulation is in mixed strategies with a support containing two distinct levels, m_i' and m_i''. Suppose that there is such an equilibrium in which the states go to war with probability 0. This means that on the path an offer is accepted with probability 1 (and we can select m_i' and m_i'' such that war occurs with probability 0 following these two accumulation levels). Since both m_i' and m_i'' are in the support of i's equilibrium strategy, we must have $v_i' - v_i'' = \beta_i(m_i' - m_i'')$, where v_i' and v_i'' denote the expected discounted payoffs to the equilibrium settlement when i selects m_i' and m_i'', respectively. This means that $v_i' \neq v_i''$.

Step 3: Now let $\xi_i(m_i, \sigma_i)$ and $\xi_j(m_j, \sigma_j)$ denote the mappings from choices in the accumulation stage into strategies in the bargaining protocol. With this notation, we can express equilibrium expected discounted payoffs from the bargaining settlement as functions of the form $v_i(\xi_i(m_i, \sigma_i), \xi_j(m_j, \sigma_j))$ and $v_j(\xi_i(m_i, \sigma_i), \xi_j(m_j, \sigma_j))$. Since war occurs with probability 0 on the path following both m_i' and m_i'' it cannot be the case that $v_i(\xi_i(m_i', \sigma_i), \xi_j(m_j, \sigma_j)) \neq v_i(\xi_i(m_i'', \sigma_i), \xi_j(m_j, \sigma_j))$. If this inequality held, say with the former larger than the latter, then player i would have an incentive to deviate (say by playing $\xi_i(m_i', \sigma_i)$ when $m_i = m_i''$). Recall that state j would not know that i had deviated (because m_i is hidden and and the support of $F(\sigma_j|m_i')$ coincides with the support of $F(\sigma_j|m_i'')$, and thus player j's bargaining behavior could not respond to the deviation. Thus, we have shown that $v_i(\xi_i(m_i', \sigma_i), \xi_j(m_j, \sigma_j)) = v_i(\xi_i(m_i'', \sigma_i), \xi_j(m_j, \sigma_j))$.

[16] To be clear, no delay means that following any profile of accumulations m and messages that are possible in equilibrium, the resuling bargaining strategies reach a settlement before any discounting occurs.

Step 4: But since $v_i' \neq v_i''$, it must be the case that $\xi_j(m_j, \sigma_j') \neq \xi_j(m_j, \sigma_j'')$ for some realizations σ_j' and σ_j''. Now since i does not observe σ_j and since σ_i and σ_j are independent (by construction the mixtures that players use are independent and thus the signals are unconditionally independent) it cannot be the case that $v_j(\xi_i(m_i, \sigma_i), \xi_j(m_j, \sigma_j')) \neq v_j(\xi_i(m_i, \sigma_i), \xi_j(m_j, \sigma_j''))$ for any values of m_i, σ_j, m_j. This is true because if the inequality held, j would have an incentive to deviate (this can be shown by using a very similar argument). So, holding fixed m_j, the expected discounted payoff from bargaining to j cannot be different under m_i' and m_i''. But since war happens with probability 0, the assertion that $v_i' \neq v_i''$ contradicts the assumption that equilibrium settlements sum to 1 and the assumption that $\delta = 1$ or the equilibrium involves no delay. ■

We also note that the necessary and sufficient conditions for the existence of the unarmed, certain-peace equilibrium are exactly as given in Proposition 5. To see this result, remember that in an equilibrium with $\mathbf{m} = \mathbf{0}$, since $F(\cdot|m_j)$ has full support for all m_j and the equilibrium is in pure strategies, $\mathbf{m} = \mathbf{0}$ remains known after the signals are observed. The proof then proceeds exactly as given in the previous section. (For the proof of part (ii) with the added noisy signals, note that any deviation in which states reach a settlement with probability one and involves $m_i > 0$ could be improved on by a strategy that has $m_i = 0$ and mimics this deviation because j will believe $m_i = 0$ after any signal it receives.)

Without discounting, we again rule out equilibria in which states act unpredictably and keep secrets about their military acquisitions, thus making their adversaries uncertain about their overall levels of military capacity, but never go to war. With discounting, we cannot rule out such equilibria. However, because we have put so few restrictions on the payoffs from war, our results apply very generally, even to situations that one might consider to involve discounting. For example, a situation in which a state must engage in time-consuming mobilization before starting a war can be represented as one in which the state has a lower (expected) value for war instead of as one with discounting; in this case, our results apply.

The existence and precise form of any equilibria in which states act unpredictably but always remain at peace depend on the particular bargaining protocol. However, we can make several observations about their features. First, if an equilibrium of this form exists, it must involve states taking some time to reach a bargain (delay). Second, a state's (i's) payoffs cannot be affected by the signal σ_i that it observes about the adversary's capabilities; if it were, the state always would pretend to observe the signal that gave it the best payoff. Thus, in such an equilibrium, the noisy signals are not valuable. This finding leads to an interesting conclusion. States would not be willing to devote resources to see a noisy signal of the form in this model in any equilibrium in which war occurs with probability 0. Thus, while we have not extended Proposition 4 to the case of noisy signals, equilibria that involve war with probability 0 but nondeterministic arming require that states can observe signals that are meaningless to them (in the sense that what a state learns about an opponent's strength does not affect its payoff). This finding leads us to conclude that a satisfactory explanation of arms accumulation and bargaining that involves uncertainty and costly spying must also involve a risk of war.

Third, if an equilibrium in which states act unpredictably but never go to war exists, the payoff of any state i that generates uncertainty must be affected by the signal that its adversary observes about its capacity, σ_j. For a state to be willing to create uncertainty about its payoffs, it must expect a better bargain, on average, when it has invested more in military capacity; thus, the adversary's actions must be tied to the signals it receives. The second and third points together imply that an equilibrium of the noisy signals game with uncertainty and without war must take a particular form: The signal a state receives about its adversary must affect the adversary's expected payoff without affecting its own. This is only possible if the probability that a settlement occurs in a given period is affected by the signals.

Thus, without discounting, our results about the difficulties of avoiding secrets and war extend to a version of the model that includes noisy signals about the states' military accumulations — even when those signals are just a little bit noisy. Even with some ability to observe each other's accumulations, states act somewhat unpredictably and keep secrets about their military acquisitions in every equilibrium except the one in which they remain disarmed, or armed at some known, status-quo level. In addition, a certain-peace equilibrium exists only as long as each state's net payoff to secretly arming as much as it likes and starting a war is less than its payoff from the status-quo bargain. With discounting, we do not rule out the possibility of a certain-peace equilibrium in which states create uncertainty about their military capacities. Such an equilibrium only can exist, however, when it takes time to reach a bargain, and when (in equilibrium) getting information about an adversary's military capacity is not valuable.

CONCLUSION

Much of the literature on war and on international institutions rests on the assumption that countries have private information about their military capabilities. Yet, military capabilities are not simply an extant feature of the world; they are the result of choices made by states, or by their leaders. This paper helps to explain war by explaining why states arm in such a way as to create uncertainty about their military capabilities — that is, why they build their military strength in such a way that the overall level is not completely predictable, and hide information about both their budgets and particular programs.

We have presented a simple model in which states acquire military capacity, bargain, and go to war only if they fail to reach an agreement. The model has no *ex ante* private information. If states maintain a situation of full information (their investments in military capacity are in pure strategies or they choose to reveal their military capacity), there is always a bargain they prefer to war. Yet, unless there is no incentive to arm unilaterally in secret and start a war, states arm in a way that makes adversaries uncertain about their capabilities and go to war with positive probability in every equilibrium of our games. States behave this way because of competing incentives — the incentive to prevail in war, and the incentive to minimize military expenditures. These conclusions are quite robust; they follow from a large class of bargaining models, and from an alternative version of the model in which states get noisy signals about their adversaries' military acquisitions.

Taking our model literally, the conditions for the equilibrium in which states maintain complete information and never go to war seem unlikely to be satisfied often in practice. If states begin their interaction with no arms, then the certain-peace equilibrium exists only if it is better for each state to have the bargain reached when both are unarmed than to be a superpower at war with an unarmed adversary. Assuming instead that states begin with some known level of armaments, our conclusions are somewhat more optimistic; certain peace exists if there is no incentive to arm in secret, unilaterally and perhaps heavily and then to start a war, relative to remaining at some status-quo bargain based on the *ex ante* distribution of military power. However, certain peace is less likely when the costs of arming is lower, so that positive externalities of arming make it more likely that states create uncertainty and go to war with positive probability.

The drawback of the generality of our models is that we are not able to derive testable propositions. As Kennan and Wilson (1993) explain, the detailed empirical implications of bargaining models generally vary with the particulars of the bargaining protocol. We do not restrict our model to a particular protocol because states use a wide range of bargaining procedures when negotiating and we want our results to be widely applicable. For a particular international interaction, one could determine a reasonable protocol and payoffs from war, and then derive testable propositions. Such an exercise remains for future work.

While the substantive conclusions of the analysis are negative in the sense that certain-peace equilibria exist only in circumstances that seem to us to be empirically rare, one important positive interpretation should be noted. The model and analysis broaden the scope of rationalist studies of security. By moving beyond a taxonomy of the types of asymmetric information and explaining the emergence of this critical feature of the international landscape, the model provides a framework from which future studies may learn how bargaining procedures, bilateral agreements, and international institutions can limit the scope of costly asymmetric information and investment in military capacity.

In the introduction to this paper, we noted that international institutions often are thought to reduce the likelihood of war in part by providing information to states and lessening the extent of harmful uncertainty. Our paper points to unanswered questions about international institutions. We find that desirable equilibria in which states reveal information and do not go to war are possible when the institution allows states to demonstrate their military capabilities perfectly (with no noise or possibility of cheating). Regrettably, this conclusion hinges on the term "perfectly;" even a minuscule amount of noise (or doubt on the part of a state receiving a signal) undermines the effectiveness of the institutions. We might think of this conclusion as saying that "close-to-perfect" signals and information are not good enough. An important caveat is that our focus on generality has come at a price. We have not characterized the equilibrium likelihood of war in these settings. It remains possible that the "closer-to-perfect" the signals are, the closer to peace the international order remains. We leave this investigation to future work.

Finally, states in our model begin with no private information but have incentives to acquire it. The existing literature does not explain how international institutions can overcome states' incentives to acquire additional uncertainty once the institutions (hypothetically) have revealed all existing uncertainty. Thus, a logical next step is to investigate whether any voluntary monitoring schemes beyond those we consider here can ameliorate the incentives for arming and secrecy. Overall, we hope that a broader investigation of international organizations will help explain what types of mechanisms can reduce the likelihood of military conflict.

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